Power Delay Profile Estimation Technique for LMMSE Channel Estimator of MIMO-OFDM System

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Abstract- A multiple-input multiple-output (MIMO) communication system combined with the orthogonal frequency division multiplexing (OFDM) modulation technique can achieve reliable high data rate transmission over broadband wireless channels. But the performance gain depends heavily on accurate channel estimation. In linear-minimum-mean-Square error (LMMSE) channel estimation for multicarrier system, one needs to know the channel statistics. We propose a power delay profile (PDP) estimation technique to obtain the frequency domain channel statistics at the receiver. The pilot symbols of all transmit antenna ports are used in estimating the PDP. The estimated PDP is used to generate the LMMSE filter Coefficients for data-subcarrier channel estimation. The distortions caused by null subcarriers and an insufficient number of samples for PDP estimation are also considered. The proposed technique effectively reduces the distortions for accurate PDP estimation. Simulation results show that the performance of LMMSE channel estimation using the proposed PDP estimate approaches that of Wiener filtering due to the mitigation of distortion effects.

Index Terms- Channel Estimation; PDP; MIMO; OFDM; LMMSE; 3GPP-LTE;

I. INTRODUCTION

Telecommunications in the current information age is increasingly relying on the wireless link. This is because wireless communication has made possible a variety of services ranging from voice to data and now to multimedia. Consequently, demand for new wireless capacity is growing rapidly at a very alarming rate. In a bid to cope with challenges of increasing demand for higher data rate, better quality of service, and higher network capacity, there is a migration from Single Input Single Output (SISO) antenna technology to a more promising Multiple Input Multiple Output (MIMO) antenna technology. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) technique has emerged as a very popular multi-carrier modulation technique to combat the problems associated with physical properties of the wireless channels such as multipath fading, dispersion, and interference. The combination of MIMO technology with OFDM techniques, known as MIMO-OFDM Systems, is considered as a promising solution to enhance the data rate of future broadband wireless communication Systems. However, this enhancement requires accurate and computationally efficient channel state information to achieve the maximum diversity or multiplexing gain [5]-[7].

The pilot-aided channel estimation, based on the linear minimum mean square error (LMMSE) technique, is optimum in the sense of minimizing mean square error (MSE) when the receiver knows the channel statistics. To obtain the frequency domain channel statistics at the receiver, power delay profile (PDP) estimation schemes have been proposed. These schemes are based on the maximum likelihood (ML) estimation by taking advantage of the cyclic prefix (CP) segment of OFDM symbols. However, the ML PDP estimators require very high computational complexity for obtaining an accurate PDP. Another approach is by employing an approximated PDP (i.e., uniform or exponential model) with the estimation of second-order channel statistics. But the performance of LMMSE channel estimator with the approximated PDP degrades due to correlation mismatch and the estimation error of delay parameters. To reduce the mismatch in the frequency domain, PDP is estimated using only the pilot symbols of all transmit antenna ports with low computational complexity. This technique effectively mitigates the distortion effects, incurred by null subcarriers and an insufficient number of estimated channel impulse response (CIR) samples. The performance of LMMSE channel estimation with the proposed PDP estimate approaches that of Wiener filtering. The performance of the estimators is presented in terms of the mean square error (MSE) and pilot SNR.
II. SYSTEM MODEL

The MIMO-OFDM system with P transmit and Q receive antennas, and K total subcarriers is considered. Suppose that the MIMO-OFDM system transmits subcarriers at the central spectrum assigned for data and pilots with K−Kq virtual subcarriers, in order to control interferences with other systems. The CIRs corresponding to different transmit and receive antennas in MIMO systems usually have the same PDP [12].

Let us consider the pilot subcarriers used to be a QPSK modulated signal from known sequences between the transmitter and receiver which is represented as CP [Kp, np] for pth transmit antenna at np th OFDM symbol. Fig.2 shows the pilot symbol arrangement in a physical resource block (PRB), that are distributed over a time and frequency grid, to retain its orthogonality among different transmit and receive antennas. Kp∈Fp and np∈T represent the index sets for the pilot subcarriers of the pth antenna port in the frequency and time domains, respectively. At the np th OFDM symbol, the number of pilot subcarriers is defined as Kp = Fp/. The pilot inserted OFDM symbol is transmitted over the wireless channel after performing an inverse fast Fourier transform (IFFT) and adding a CP. It is assumed that the length of CP, Lg, is longer than the channel maximum delay, Lc, thus the channel matrix circular (Lc ≤ Lg). The received pilot symbol for the qth receive antenna after perfect synchronization, the removal of CP, and FFT operation can be represented as,

\[ y_q[n_p] = \text{diag}(n_p) F_p h_q n_q \]

(1)

where \( x_p \) is a pilot vector at the np th OFDM symbol for \( i_k \in F_p \) and \( k=1,2,..,Kp \) and is given as \( x_p = [c_p, i_0, np, c_p, 2, np, ... , c_p, Kp, np]^T \). \( \text{diag}(x_p) \) is the \( X \times Kp \) diagonal matrix whose entries are \( Kp \) elements of the vector \( x_p \). \( F_p \) is a \( Kp \times Lg \) matrix with the \( (i_k, l) \)th entry and is given as \([F_p]_{i_k,l} = 1/|K| \exp\{-j2\pi i_k l/K\} \) where \( i_k \in F_p \) and \( l = 0,1,2,..,Lg-1 \). \( h_{p,q} \) is the LgX1 CIR matrix at the pth transmit antenna and qth receive antenna and is given as \( h_{p,q} = [h_{p,q}[n_p,0]], h_{p,q}[n_p,1], ..., h_{p,q}[n_p,Lg-1,0]^T \). \([\cdot]^T\) represents the transpose operation. \( np \) is a complex Additive White Gaussian Noise (AWGN) vector at the qth receive antenna with each entry having a zero mean and variance of \( \sigma_n^2 \). The average power of \( h_{p,q} \) depend on the delay profiles of the wireless channels.

III. PDP ESTIMATION

In this section, we will present derivation of PDP in MIMO-OFDM systems, including PDP estimation in practical MIMO-OFDM Systems. Then, the estimated PDP is used to obtain the frequency-domain channel correlation in the LMMSE channel estimator. Using the regularized least squares (RLS) channel estimation the channel impulse response can be estimated approximately from (1), at (p,q)th antenna port with a fixed length of Lg as

\[ \hat{h}_{R,p,q} = (F_p^H F_p + \varepsilon I_{Lg})^{-1} F_p^H \text{diag}(x_p)^H y_q[n_p] \]

(2)

Where \((F_p^H F_p + \varepsilon I_{Lg})^{-1} F_p^H \text{diag}(x_p)^H = w_{RLS,p} \) and \( \varepsilon = 0.001 \) is a small regularization parameter and \( I_{Lg} \) is the \( Lg \times Lg \) identity matrix. \( F_p^H F_p \) in (2) is ill-conditioned due to the sparsity of pilot tones in the frequency domain and the presence of virtual subcarriers [8]. To derive the PDP from the estimated CIR in (2), the ensemble average of \( \hat{h}_{R,p,q}^H \hat{h}_{R,p,q} \) is

\[ E\{\hat{h}_{R,p,q}^H \hat{h}_{R,p,q}\} = W_{RLS}^H + \sigma_n^2 w_{RLS,p} w_{RLS,p}^H \]

(3)

Where \( W_{RLS} = E\{h_{p,q} h_{p,q}^H\} \) and \( W = (F_p^H F_p + \varepsilon I_{Lg})^{-1} F_p^H F_p \). The PDP of multipath channel within the length of \( L_g \) is given by the diagonal elements of the channel covariance matrix, \( R_{hh} \), and all off-diagonal elements are zeros. Hence, the covariance matrix can be expressed as.

\[ R_{hh} = \text{diag}(p_h) \]

(4)

Where \( P_h = [p_0, p_1, ..., p_{Lg-1}, 0, ..., 0]^T \) and \( p_i = |h_{p,q}[n_p, l]|^2 \). Due to the presence matrix \( F_p^H F_p \) is ill conditioned. So \( R_{hh} \) is distorted by \( W \), thus we investigate the method for eliminating the spectral leakage of \( W \). The covariance matrix of the estimated CIR is defined as

\[ R_{\text{dd}} = \text{w} \text{R}_{hh} W^H = \sum_{i=0}^{Lg-1} \text{w} \text{diag}(p_i) u_i^H \]

(5)

Where \( u_i \) is a unit vector with the \( i \)th entry being one and otherwise zeros. Let \( P_S = D_g (R_{\text{dd}}) \) and \( t_i = D_g (W \text{diag}(u_i) W^H) \) where \( D_g (A) \) is the column vector containing all the diagonal elements of \( A \).

Then, the relation is simplified as

\[ P_h = p_0 f_0 + p_1 t_1 + ... + p_{Lg-1} t_{Lg-1} \pm T_h \]

(6)

Where \( T = [t_0, t_1, ..., t_{Lg-1}] \) is defined as a distortion matrix by \( W \). It is noted that the distortion matrix is a strictly diagonally dominant matrix since the non-diagonal elements of \( T \) are composed of the leakage powers of \( u_i \) for all \( i \). From the Gregorian circle theorem, a strictly diagonally dominant matrix is nonsingular [13]. In addition, the distortion matrix is a well-conditioned matrix. Hence, the distortion of \( W \) can be eliminated as \( P_h = T^{-1} P_S = E\{g_{p,q}[n_p]\} - \sigma_n^2 \tilde{W} \)

(7)

Where \( g_{p,q}[n_p] = T^{-1} D_g(h_{R,p,q}) \hat{h}_{R,p,q}^H \) is defined as the received sample vector for estimating PDP at the (p,q)th antenna port on the np th OFDM symbol, and \( \tilde{W} = T^{-1} D_g(\sigma_n^2 w_{RLS,p} w_{RLS,p}^H) \).
A. Proposed PDP Estimation technique

The received sample vector in (7) can be expressed as

$$g_{p,q}[n_p] = D_g(h_{p,q} h_{p,q}^H) + D_{p,q} + e_{p,q}$$  \hspace{1cm} (8).$$

Where $D_{p,q} = T^{-1} D g(w_{RS,p} n_q w_{RS,q}^H)$ and $e_{p,q} = 2 \text{Re}(T^{-1} D g(W_h p_q n_q w_{RS,q}^H))$. We assume that

$D_{p,q}$ is an effective noise by AWGN. Then, the sample average of $g_{p,q}[n_p]$ is given by

$$< g_{p,q}[n_p] >_{N} = \sum_{p,q=1}^{Q} g_{p,q}[n_p]$$  \hspace{1cm} (9).$$

Where $N = |T_p| P Q$ represents the total number of samples for PDP estimation. $|T_p|$ is the number of pilot symbols at the Kth subcarrier in a time slot. When N is sufficiently large, the PDP can be perfectly estimated, since $D_g(h_{p,q} h_{p,q}^H) \rightarrow \hat{W}$, and $e_{p,q} \rightarrow 0$. However, it is difficult for a receiver of practical MIMO-OFDM systems to obtain such a large number of samples. With an insufficient number of samples, the PDP can be approximated as

$$D_g(h_{p,q} h_{p,q}^H) \approx D_g(h_{p,q} h_{p,q}^H) + e_{p,q}$$  \hspace{1cm} (10).$$

To improve the accuracy of PDP estimation with insufficient samples, we mitigate the effective noise as follows

$$D_g(h_{p,q} h_{p,q}^H) + e_{p,q} = \hat{D_g}(h_{p,q} h_{p,q}^H)$$  \hspace{1cm} (11).$$

Where $Z_N = g_{p,q}[n_p] >_{N} - \sigma^2_N \hat{W} >_{N} + Z_N$. Since $|P_c| \geq 0$ for all i, the PDP can be initially estimated as

$$P_{init} = \frac{1}{N} \sum_{p=1}^{N} \sum_{q=1}^{Q} g_{p,q}[n_p]$$  \hspace{1cm} (12).$$

Where $S_{p,q}[n_p]$ is the sample vector of proposed PDP estimator with the lth entry

$$S_{p,q}[n_p] = \begin{cases} 0 & \text{if } \hat{g}_{p,q}[n_p] \leq \sigma^2_N \hat{W}^l \\ \hat{g}_{p,q}[n_p] - \sigma^2_N \hat{W}^l & \text{otherwise} \end{cases}$$  \hspace{1cm} (13).$$

Where $\hat{g}_{p,q}[n_p] = [g_{p,q}[n_p]]_l$ and $\hat{W}^l = [\hat{W}]_l$. To mitigate the detrimental effect of residual noise $Z_N$, the proposed scheme estimates the average of residual noise at the zero-taps of $P_{init}$. Then, the estimated PDP in (16) can be used to obtain the frequency-domain channel correlation in the LMMSE channel estimator.

IV. PERFORMANCE AND COMPLEXITY ANALYSIS

The LMMSE channel estimator with the imperfect PDP in (16) is given by

$$w_{f,p} = F_p D g(P_{init}) F_p^H (F_p \hat{P}_h F_p^H + \sigma^2 N I_{k_p})^{-1}$$  \hspace{1cm} (17).$$

Where $F_{L}$ is the KqXLq matrix obtained by taking the first Lq columns of the DFT matrix. $P_{init} = P_h + e_{pdp}$ expressed as the estimated PDP, where the lth element of $e_{pdp}$ is defined as

$$e_{pdp}^l = \begin{cases} [\hat{N}]_l - \hat{N}_{avg} & \text{if } [\hat{N}]_l > \hat{N}_{avg} \\ 0 & \text{if } [\hat{N}]_l \leq \hat{N}_{avg} \end{cases}$$  \hspace{1cm} (18).$$

The error covariance matrix of LMMSE channel estimation with the imperfect PDP can be obtained as

$$E_p = E \{ (F_{L} h_{p,q} q - W_f, p_h L S, p_q) (F_{L} h_{p,q} q - W_f, p_h L S, p_q)^H \}$$  \hspace{1cm} (20).$$

Where $\beta_{th} = \frac{1}{L_p} \sum_{l=0}^{L_p} P_{init}^l$ is defined as a threshold value for the zero-tap detection. Then, the average of residual noise at the zero-taps can be estimated as

$$\hat{N}_{avg} = \frac{1}{N} \sum_{l=0}^{L_p} P_{init}^l$$  \hspace{1cm} (21).$$

Then, the estimated PDP in (16) can be used to obtain the frequency-domain channel correlation in the LMMSE channel estimator.
Using the error covariance matrix, the frequency domain MSE of the proposed scheme is given by

\[
\text{MSE}_{F_p} = \text{Tr}(\mathbf{E}_P)/\text{Tr}(\mathbf{F}_L \mathbf{D} \mathbf{g}(P_s) \mathbf{F}_L^H)
\]

(21)

Where \( \text{Tr}(\mathbf{E}_P) \) denotes the trace operation of \( \mathbf{E}_P \). With a sufficiently large number of samples, \( \hat{\mathbf{E}}_{pdp} \to 0 \). Thus, the MSE of the proposed scheme achieves that of Wiener filtering because \( \mathbf{W}_f \rightarrow \mathbf{W}_{opt,p} \).

The additional complexity by the proposed PDP estimation technique is \( O(\|T_p\| PQL_g) \), which mainly comes from computing (2) and (4). When the pilot spacing is fixed in the frequency domain, all entries of \( \mathbf{F}_p \) and \( \mathbf{T} \) are constant. Thus, \( (\mathbf{F}_p^H \mathbf{F}_p + \mathbf{I}_{L_g})^{-1} \mathbf{F}_p^H \) and \( \mathbf{T} \) inverse can be computed only once, and their values can be stored. The additional complexity is then reduced to \( O(L_g^2 + \|T_p\| PQL_g) \).

V. SIMULATION RESULTS

We consider a MIMO-OFDM system with the physical layer parameters for the downlink of 3GPP LTE [14]. The system bandwidth is 5 MHz with 301 subcarriers for transmitting data information and pilots including a DC subcarrier at 2 GHz carrier frequency. The width of each subcarrier is 15 kHz with an FFT size of 512. The MIMO OFDM system utilizes four transmit and two receive antennas (\( P=4, Q=2 \)). We assume that the pilots of the four transmit antenna ports are distributed as the time and frequency grid of the LTE system in Fig. 2. The length of CP is 40 (\( L_g=40 \)). For all simulations, the channel estimator is based on a cascaded 2×1D LMMSE technique during 14 OFDM symbols (\( |T_1| = |T_2| = 2, |T_3| = |T_4| = 1 \)), as shown in Fig. 2, where the filtering in frequency domain is followed by the filtering in time domain over slowly fading channels with the Doppler frequency of 5 Hz.

Figure 2 shows the MSE performance of the 2×1D LMMSE technique using the estimated PDP. All underlying links are modelled as extended typical urban (ETU) channels. The performance of the 2×1D Wiener filter with exact PDP is included as a lower bound. For performance comparisons, we plot the performance of frequency domain regularized LS channel estimation in which the PDP information is not required. The performance of the 2×1D LMMSE technique using the approximated PDP, which is uniform or exponential model with the channel delay parameter estimation in [10], is also plotted. Note that the LMMSE technique using the estimated PDP outperforms the conventional methods, since the correlation mismatch is reduced by the proposed PDP estimation. We also observe from Fig. 4 that the proposed method has a performance loss within only a 2.4-dB gap, compared with 2×1D Wiener filtering.

**Fig.3. Performance of LMMSE with estimated PDP technique with different mobile equipment speeds**

Figure 3 shows the MSE performance of the 2 X 1D LMMSE technique using the estimated PDP for different mobile equipment speeds at 30-dB SNR. All underlying links are modelled as ETU channels. In Fig. 3, it can be seen that the MSE of LMMSE technique using the estimated PDP achieves that of Wiener filtering even at high Doppler frequencies. In Fig. 4, we investigate the MSE performance of the proposed scheme over the exponentially power decaying six-path Rayleigh fading channel model, where the channel maximum delay, \( L_{ch} \), is variable. The PDP of the channel model is defined as

\[
E\{ |a_{p,q}^{2} | \} = \frac{1}{s_h} e^{1/\tau_{rms}} \quad \text{for} \quad l=0, \Delta \tau_{ch}, ..., 5\Delta \tau_{ch} \quad \Delta \tau_{ch} \quad \text{variable}.
\]

The PDP is exponentially decaying in time domain, \( \tau_{rms} = \frac{L_{ch}}{\log(L_{ch})} \) and \( s_h \) is the normalization factor. The performance of the proposed scheme is better than that of the conventional methods, and approaches that of Wiener filtering in various channel environments.
VI. CONCLUSION

We proposed a PDP estimation technique for the LMMSE channel estimator in MIMO-OFDM systems. The CIR estimates at each path of the MIMO channels were used to obtain the PDP. For accurate PDP estimation, we considered the spectral leakage effect from virtual subcarriers, and the residual noise caused by the insufficient number of estimated CIR samples. The proposed technique effectively mitigates both the spectrum leakage and residual noise. Simulation results show that the performance of LMMSE channel estimation using the proposed PDP estimate approaches that of Wiener filtering due to the mitigation of distortion effects.

VII. REFERENCES

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