An Approach for Developing an Optimum Quantity Discount Policy of Deteriorating Items Inventory Transportation System

BANI MUKHERJEE
Professor, Department of Applied Mathematics,
Indian Institute of Technology (I S M),
Dhanbad-826004, Jharkhand, India

AANCHAL BANSAL
Student, Int M. Tech (M&C), Department of Applied Mathematics, Indian Institute of Technology (Indian School of Mines), Dhanbad-826004, Jharkhand, India

Abstract—A deteriorating item inventory model under Weibull pattern deterioration rate with optimal quantity discount policy and mixed cargo transportation modes has been studied. The transportation modes are full container load (FCL) and less than container load (LCL). Deteriorating items, such as specialty gases which are applied in semiconductor fabrication etc. deteriorate owing to environmental variation. A quantity discount problem has been discussed between a seller (wholesaler) and a buyer (retailer). The seller purchases products from an upper-leveled supplier (manufacturer) and then sells them to the buyer who faces customers’ demand. The seller attempts to increase profit by controlling the buyer’s order quantity through a quantity discount strategy and the buyer tries to maximize profit considering the seller’s proposal. In the proposed model using the differential equations under various cost considerations, expressions for total cost function per unit time; optimum inventory level and optimal length of a cycle are obtained. All the expressions are illustrated graphically with the help of numerical example. The sensitivity analysis for the model has been performed to study the effect on cost function due to change of the values of the parameters associated with the model.

Keywords: Inventory; Quantity Discount; Deteriorating Items; Transportation Costs;

1. INTRODUCTION

Inventory is the stored resource of either raw material or work-in-process or finished goods, that is used to satisfy present or future demand. In general the purposes of inventory system studies includes either Smooth-out variations in operation performances or avoid stock out or shortage or safeguard against price changes and inflation or take advantage of quantity discounts. It is a widespread economic phenomenon that the price of a good depends among many other things – one of this is the amount ordered. Indeed, there are many reasons for suppliers to offer discounts based on the volume sold to a buyer. Consequently, when it comes to procuring amounts of different goods from different suppliers, it makes sense to consider various alternatives. In fact, choosing the right suppliers to deliver the right products has become a major concern in many large companies. Reliability, quality and price are important criteria that guide the choice for suppliers. Moreover, the ever-increasing opportunities that e-commerce and web-based procurement offer for dealing with procurement issues, explain the increased usage of so-called reverse auctions. While traditional auctions involve a single seller and multiple buyers, a reverse auction involves multiple sellers that express bids to provide goods or services and one buyer that chooses the best bids.

Davenport and Kalagnanam [1] report on a volume discount auction in which discounts are based on quantities for each individual good. Furthermore, they use an incremental discount policy, meaning that the discounts apply only to the additional units above the threshold of the volume interval. Hohner et al. [2] describe a web-based implementation of this procurement auction at Mars Incorporated. Eso et al. [3], also elaborate on the work of Davenport and Kalagnanam. They study a volume discount auction with piece-wise linear supply curves, allowing discontinuities and all-unit discounts. However, they do require additive separable supply curves, which boils down to assuming that the prices charged by a supplier for different commodities are independent. This makes their problem not truly combinatorial, since synergies or substitutability between different goods cannot be reflected in the total price charged by the suppliers. As a result, a total quantity discount structure is not possible in their setting. The authors formulate a column generation based heuristic that provides near-optimal solutions to the bid evaluation problem. Another procurement auction with marginal-decreasing piecewise-constant supply curves is described in Kothari et al. [4]. This auction also allows all-unit discounts, but it deals only with a single good. Kothari et al. present fully polynomial-time approximation schemes for the winner determination problem and the computation of the Vickrey-Clarke-Groves payments of this auction.

The status of inventory problems faced by the wholesaler and business, have been changed a lot in present situation. Now-a-days, due to globalization and appearance of multi-nationals,
the interest of both the wholesaler and their franchisees may be considered as retailers, have been coupled together and the objective of both the parties are to be satisfied simultaneously. Hence all these considerations normally lead to the formulation of multi-objective decision making (MODM) inventory control problem. Though MODM problems have been formulated and solved in many other areas like air pollution, structural analysis etc., till now only a few papers on MODM have been published in the field of inventory control. Padmanabhan and Vrat [5] formulated an inventory problem of deteriorating items with two objectives- minimization of total average cost and wastage cost in crisp environment and solved by non-linear goal programming method. Roy and Maiti [6] formulated an inventory problem of deteriorating items with two objectives, namely, maximization of total average profit and minimization of total wastage cost in fuzzy environment.

In many real-life situations, retailers deal with perishable products such as fresh fruits, food-stuffs and vegetables. The inventory of these products is depleted not only by demand but also deterioration. Yang [7] has developed the model to determine an optimal pricing and a ordering policy for deteriorating items with quantity discount which is offered by the vendor. The rate of deterioration is very small in some items like hardware, glassware, toys and steel that there is little need for considering deterioration in the determination of the economic lot size whereas items such as fish, medicine, vegetables, blood, gasoline, alcohol, radioactive chemicals and food grains like wheat, potato, onion etc. have finite shelf life and deteriorate rapidly over time. The constant rate of deterioration is assumed in most researches for deteriorating inventory. But, the Weibull deterioration is used to show the product in stock deteriorates with time. Fitting empirical data in mathematical distribution shows the way to many researchers to use the Weibull distribution to model the deterioration rate. The items in which the deterioration rate follows the Weibull distribution are roasted ground coffee, corn seed, frozen foods, pasteurized milk, refrigerated meats and ice creams. An EOQ inventory model for items with Weibull distribution deterioration rate and ramp type demand was formulated by Wu [8]. Bhaskar Bhuaula, M. Rajendra Kumar, [9] Considered a problem by taking an economic order quantity model for Weibull deteriorating items with stock dependent consumption rate and shortages under Inflation. Study of few researcher on Weibull deteriorating items are namely by Zhao, Q.-Z., Zhao, P.-X [10] and Skouri, K., Konstantaras, I., Papachristos, S., Ganas, I [11].

In this paper, a deterministic deteriorating items inventory model, one wholesaler and one retailers, with mixed cargo transportation (full container load, FCL, and less than container load, LCL) over a given finite planning horizon has been developed and studied. The customer’s demand per unit time $d$ is constant and shortage is not allowed. The retailer places orders to its supplier and goods are transported by cargos. FCL and LCL cargoes are used. LCL is a shipment of cargo by which goods do not fully fill the container. The remaining space of the Less than Container may be filled with goods of other shippers. Generally, the transportation cost per unit under FCL is cheaper than LCL. Therefore, if goods can fill an entire container, FCL cargo will be the first choice.

## II. FORMULATION OF THE MODEL

A deterministic deteriorating items inventory model, one wholesaler and one retailers, with mixed cargo transportation over a given finite planning horizon has been depicted as in figure-1 and 2 respectively as follows:

![Figure-1](image1.png)

**Figure-1**

![Figure-2](image2.png)

**Figure-2**

To develop inventory model for both wholesaler and retailers, the following notations and assumptions/ criteria are used:

### A. Assumptions/ criteria

(i) There is one wholesaler and one retailers.

(ii) Lead time is zero.

(iii) Replenishment rate is infinite but replenishment size is finite.

(iv) The inventory planning horizon is infinite.

(v) The inventory system involves only one item.
(vi) Only a single order will be placed in each cycle and the entire lot is delivered in one batch.
(vii) Shortages are not allowed.
(viii) The time to deterioration of the item is distributed as Weibull ($\alpha, \beta$), that is, at time $t$, $\alpha \beta t^{\beta - 1}I(t)$ units have been deteriorated, where $\alpha, \beta$ are positive parameters.

B. Notation

- $\alpha \beta t^{\beta - 1}$: Deterioration rate
- $T$: Time period per cycle for retailer
- $T_{\text{de}}$: The time before discount
- $T_{\text{No}}$: Time period per cycle for wholesaler
- $N_{\text{o}}$: Number of cycles of retailer for per cycle of wholesaler
- $B_{\text{C}}$: Buying cost for retailer
- $c_r$: Retailer buying cost per unit item per unit time
- $Total_{\text{C}_r}$: Buying cost for retailer
- $H_{\text{C}}$: Holding cost for retailer
- $h_b$: Holding cost per unit item per unit time for retailer
- $D$: Demand
- $D_{\text{C}}$: Deterioration cost for retailer
- $DIS$: Discount
- $Q$: Inventory level per cycle of wholesaler
- $Trans_{\text{C}_r}$: Transportation cost for retailer
- $k$: Units that can be transported per cycle
- $n$: Number of Transport for whole inventory
- $ctrans$: Transportation cost per cycle
- $cff$: Transportation cost object (case) wise
- $BC_{\text{w}}$: Buying cost for wholesaler
- $c_w$: Wholesaler buying cost per unit item per unit time
- $Total_{\text{C}_w}$: Total cost for wholesaler
- $H_{\text{C}_w}$: Holding cost for wholesaler
- $h_w$: Holding cost for wholesaler per unit item per unit time
- $D_{\text{C}_w}$: Deterioration cost for wholesaler
- $Trans_{\text{C}_w}$: Transportation cost for wholesaler

III. MATHEMATICAL ANALYSIS

The inventory is depleted due to the combined effect of its demand and deterioration. The variation of inventory level $I_1(t)$ changes with respect to time $t$ due to the effects of demand as well as the effects of Weibull deterioration. If $I_1(t)$ be the on hand inventory at any time $t \geq 0$, then at time $t + \Delta t$, the inventory during the time interval $(0, T)$ will be:

$$I_1(t + \Delta t) = I_1(t) - \theta(t)I_1(t)\Delta t - D\Delta t$$

Dividing by $\Delta t$ and then taking both side limit as $\Delta t \to 0$ gives

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = -D; \quad 0 \leq t \leq T$$

with deterioration

$$\theta = \alpha \beta t^{\beta - 1}; \quad 0 < \alpha < 1, 0 < \beta \leq 1$$

The inventory level, $I_1(t)$ during time period $(0, T)$ from equation (1) is as follow:

$$I_1(t)e^{\alpha \beta t} = -\int De^{\alpha \beta t} dt + c$$

With the boundary conditions:

- $I_1(t) = Q$ at $t = 0$
- $I_1(t) = 0$ at $t = T$

The level of inventory of the wholesaler and the retailer is given as follows:

$$Inv = \left(1 + \frac{DIS}{100}\right)Q \left(e^{\alpha \beta T} - 1\right)$$

$$Q = \left(1 + \frac{DIS}{100}\right)D \left(T + \alpha \beta t^{\beta + 1} + \frac{\alpha^2 T^{2\beta + 1}}{2(\beta + 1)} + \frac{\alpha^3 T^{3\beta + 1}}{3(\beta + 1)} + \cdots \right)$$

where $T = T_{\text{int}} \left(1 + \frac{DIS}{100}\right)$

IV. CALCULATION OF COST FUNCTION

The holding cost $HC$, deteriorating cost $DC$ and transportation cost $Trans\ C$ are respectively calculated as follows:

$$\text{Holding Cost} \quad HC = h_i \int_0^T Q_i dt$$

$$\text{Deterioration Cost} \quad DC = c_i \left[Q_i - \int_0^T D_i dt\right]$$

$$\text{Transportation Cost}$$

Case ‘a’:

$$TransC = nc_{\text{trans}} + (Q - nk)c_q, nk \leq Q \leq (n + 1)k$$
**Case 'b':**

TransC = \(c_{\text{final}} + nk + U \leq Q \leq (n+1)k\)

Where, \(i = r\) for retailer and \(i = w\) for wholesaler

**Case 1. Retailer**

**Buying cost**

\[ BC_r = c_r \cdot Q \]

where the Retailer buying cost per unit item

\[ c_r = \left(1 - DIS_{\text{100}}\right) \cdot 400 \]

**Holding cost**

\[ HC_r = n \cdot Q \cdot e^{\alpha r} \cdot \frac{\left(e^{\beta r} - 1\right)}{\alpha} \cdot \frac{\left(e^{\beta r} - 1\right)}{\alpha} + \frac{Q}{T} - TD \]

Deterioration cost 

\[ DC_r = c_r \cdot (Q - DT) \]

**Transportation cost**

\[ n = \frac{Q}{k} \]

1) **Case i**: less than container load (LCL)

\[ TransC_r = n \cdot c_{\text{trans}} + (Q - nk) \cdot ctf \]

2) **Case ii**: Full container load (FCL)

\[ TransC_r = (n+1) \cdot c_{\text{trans}} \]

**Total cost per unit time for retailer**

\[ TotalC_r = \left(BC_r + HC_r + DC_r + TransC_r\right) \cdot No + T \]

**Case 2. Wholesaler**

**Buying cost**

\[ BC_w = c_w \cdot Inv \]

**Holding cost**

\[ HC_w = n \cdot Inv \cdot e^{\alpha r} \cdot \frac{\left(e^{\beta r} - 1\right)}{\alpha} + \frac{Q}{T} - TD \]

Deterioration cost

\[ DC_w = c_w \cdot (Inv - Q) \cdot No \]

**Transportation cost**

\[ n = \frac{Inv}{k} \]

**Case i**: less than container load (LCL)

\[ TransC_w = n \cdot c_{\text{trans}} + (Inv - n \cdot k) \cdot ctf \]

**Case ii**: Full container load (FCL)

\[ TransC_w = (n+1) \cdot c_{\text{trans}} \]

Total cost per unit time for Wholesaler

\[ TotalC_w = \left(BC_w + HC_w + DC_w + TransC_w\right) \cdot No \cdot T \]

V. **GRAPHICAL ANALYSIS**

**Case 1. Retailer**

The graphical representation of the total discount with the total cost per cycle for both the transportation modes that is case-i less than container load (LCL) and case-ii full container load (FCL) of retailer is done with the help of Mathematica software as in figure 3 and figure 4 respectively, using the numerical values

\[ \alpha = 0.01, \beta = 0.5, D = 0.04, c_h = 3, c_{\text{trans}} = 2, ctf = 0.1, n = 2, k = 5 \]

**Figure 3**

**Figure 4**

**Case 2. Wholesaler**

The graphical representation of the total discount with the total cost per cycle for both the transportation modes that is case-i less than container load (LCL) and case-ii full container load (FCL) of Wholesaler is done with the help of Mathematica software as in figure 5 and figure 6 respectively, using the numerical values

\[ ctf = 0.1, c_w = 3, \alpha = 0.01, D = 0.04, \beta = 0.5, No = 50, k = 5, c_{\text{trans}} = 2, c_w = 1 \]

**Figure 5**

**Figure 6**
VI. SENSITIVITY ANALYSIS

Case 1. Retailer

Sensitivity analysis for both the transportation modes that is case- i less than container load (LCL) and case- ii full container load (FCL) of retailer is carried out by changing a specified parameter by -50%, -25%, +25% +50% keeping the other remaining parameter at their standard value.

\[ \alpha = 0.01, \beta = 0.5, D = 0.04, c_s = 3, c_{trans} = 2, c_{tf} = 0.1, n = 2, k = 5, DIS = 90, T = 100 \]

Case i: (LCL)

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<th>-25(in %)</th>
<th>25(in %)</th>
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Case ii: (FCL)

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Case 2. Wholesaler

Sensitivity analysis for both the transportation modes that is case- i less than container load (LCL) and case- ii full container load (FCL) of wholesaler is carried out by changing a specified parameter by -50%, -25%, +25% +50% keeping the other remaining parameter at their standard value.

\[ c_{tf} = 0.1, c_s = 3, \alpha = 0.01, D = 0.04, \beta = 0.5, No = 50, k = 5, c_{trans} = 2, c_s = 1, DIS = 90, T = 100 \]

Case i: (LCL)

<table>
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Case ii: (FCL)

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VII. CONCLUSION

From above it can be said that the change in $\alpha$ and $\beta$ highly affect the total cost per unit time for both the transportation modes for Wholesaler but not for the Retailer. In both cases, with increase of DIS cost is slightly affected whereas with increase of $D$ and $T$ cost is moderately effected for both the transportation modes.

In these countries, retailers generally purchase the commodities, especially food-grains, in terms of lots (some particular amount of quantities) depending upon the capital available to them. After that they joined together to hire a truck/small transporting vehicle to transport the commodities to the selling place. The same type of procedure is followed by small retailer for purchase from retailer (country side), with the exception that here mode of transport may be bullock cart, country boat, cycle van, etc. Keeping this scenario in mind, we have formulated this problem assuming replenishment cost as a linear function of lot-size.

Big merchant $\rightarrow$ Wholesaler $\rightarrow$ retailer $\rightarrow$ Country side $\rightarrow$ small retailer $\rightarrow$ ordinary customer

This realistic assumption makes the model more practical. A similar analysis can be carried out by considering time dependent demand pattern like $D(t) = a + bt$ where $a > 0$; $b > 0$. The model developed in this paper can be enriched by extending more realistic situations, such as, multi-items and multi wholesaler or multi retailer.

VIII. REFERENCES


