Finite Element Analysis and Mathematical Calculation of Spring Back in Rotary Draw Tube Bending

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Abstract—Tube bending is a widely used manufacturing process in the aerospace, automotive, and various other industries. Cold bending of metal tubes is very important production method considering that metal tubes are widely used in a great variety of engineering products, such as automobile, aircraft, air conditioner, air compressor, exhaust systems, fluid lines. During tube bending, considerable in-plane distortion and thickness variation occurs. This paper deals with the study of Rotary Draw Tube Bending process and the Finite Element Analysis and Simulation of Rotary Draw Tube Bending of a seamless metal tube for the analysis of spring back effect. Mathematical and Experimental analysis of spring back are carried out and results are compared. By applying the theory on pure beam bending deformation, the mathematical model of spring back for pipe bending deformation is established. The Simpson's rule is applied for solving the radius in spring back. The paper also covers the modeling of rotary tube bending machine in ABAQUS, its simulation and analysis for different bending angles.

Keywords—Rotary Draw Tube Bending; Springback; Simulation On ABAQUS; Mathematical Calculation

I. INTRODUCTION

Rotary draw tube bending is the most flexible bending method and is used immensely in industry on account of its tooling and low cost. Rotary draw bending consists of a bend die, Clamp die, Pressure die and wiper die. In this bending technique the tube is securely clamped to the bend die by using the clamp die. The bend die rotates and draws the tube along with it. The pressure die prevents the tube from rotating along with the bend die. The pressure die may be stationary or it may move along with tube. The pressure die provides a boost (pushes the material at the extrados of the tube) to reduce the thinning of the tube and can be very helpful when the bending angle is large and the bending radius is small. A mandrel along with wiper die may be used to prevent the wrinkling and collapse of the tube. But the use of mandrel should be avoided if possible since it increases the production cost. Fig. 1 shows the tooling of rotary draw bending process. Rotary Draw Tube bending provides close control of metal flow necessary for small radius and thin walled tube, as in [1].

Fig. 1. Schematic of Rotary Draw Tube Bending

Fig. 2. Working of a Rotary Draw Tube Bender

II. DEFECTS IN TUBE BENDING

During the bending process the tube undergoes considerable in-plane distortion. The limitations in the tube bending process are distortion of cross-
section, wrinkling and variation in wall thickness, spring back and fracture

A. **Variation in wall thickness**

During the bending process the bending moment induces axial forces in the inner and outer fibers. The inner and outer fibers are subjected to compressive and tensile stresses respectively. This results in thinning of the tube wall at the outer section (extrados) and thickening of the tube wall at the inner section (intrados). The wall thickness variation is shown in Fig. 3.

![Fig. 3. Variation in Wall Thickness](image)

B. **Bursting or Fracture**

The fibers at the extrados are subjected to tensile stress. When the tensile stress induced in the tube due to the bending moment at the extrados exceeds the ultimate yield strength of the material, the tube fractures at the extrados.

C. **Wrinkling**

As the tube is bent, the inner surface of the tube, the intrados is subjected to compressive stress. When the tube is bent into a tight radius, it is subjected to high compressive stress in the intrados that leads to Bifurcation instability or buckling (wrinkling) of the tube. Wrinkles are wavy types of surface distortions. As tubes are used as parts in many applications where tight dimensional tolerances are desired, wrinkles are unacceptable and should be eliminated. Furthermore, wrinkles spoil the aesthetic appearance of the tube. Wrinkling is shown in Fig. 4.

![Fig. 4. Wrinkling of Tube](image)

D. **Cross-Section Distortion**

As described above the outer fibers of the tube are subjected to tensile stress whereas inner fibers of the tube are subjected to compressive stress. There is a tendency of fibers at both the ends to move towards the neutral axis. The outer fiber of the tube tends to move towards the neutral plane to reduce the tensile elongation. This results in the cross section of the tube being no longer circular, instead becoming oval. The common practice in industry is to provide support to the tube from inside to prevent flattening or distortion of cross section; usually a filler material or mandrel is used for that. Fig. 5 shows the cross section distortion of tube.

![Fig. 5. Cross-Section Distortion](image)

E. **Springback**

After the bending process is complete and the tooling have been withdrawn the bend tube spring backs due to the elastic nature of the tube material. This is called spring back or the elastic recovery of the tube. During the bending process internal stresses are developed in the tube and upon unloading the internal stresses do not vanish. After bending the extrados is subjected to residual tensile stress and the intrados is subjected to residual compressive stress. These residual stresses produce a net internal bending moment that causes spring back. The tube continues to spring back until the internal bending moment drops to zero. The spring back angle depends on the Bend angle; Tube material, Tube size, Mandrel, Machine and Tooling. In actual practice the amount of spring back is calculated and the tube is over bent by that amount. Fig. 6 shows spring back after the tooling has been removed.

![Fig. 6. Spring back after the Tooling is removed](image)

III. **LITERATURE REVIEW**

In the past, researchers have worked on cross section distortion, wall thickness variation, and
wrinkling issues related to pure bending of tubes. Tarana, as in [2], conducted simulations of rotary draw bending and tube hydro-forming processes. Yang et al, as in [3], simulated the rotary bending process and concluded that in the case of bending with mandrel, the section remained close to circular, but the thickness reduction at extrados can be significant. Miller et al. [4,5] conducted a series of experiments on bending of rectangular tubes on a bend-stretch form-pressure machine and developed analytical models to predict the distortion, elongation, and spring back of tubes as functions of the pressure, tension, and die radius. In their experiments they found that tubes could be formed without wrinkling at value of tension lower than the yield tension. Brazier [7] studied the distortion of round tubes in elastic bending using energy minimization. Named after Brazier’s work, the cross section deformation in tube bending is often called the Brazier effect. Zang and Yu [8] investigated the Brazier effect of an infinitely long, cylindrical tube under pure elastic-plastic bending. Vasile Adrian Ceclan Gheorghe Achima Lucian Lazarescu , Florica Mioara Groze, [22] In their paper presented a finite element model for simulation of tubes Press Bending Process. The finite element modeling is carried out on commercial code ABAQUS.

IV. TERMINOLOGY USED IN ROTARY DRAW TUBE BENDING

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>OD</td>
<td>Tube Outside Diameter. Usually specified in inches or millimeters.</td>
</tr>
<tr>
<td>ID</td>
<td>Tube Inside Diameter. Usually specified in inches or millimeters.</td>
</tr>
<tr>
<td>Extrados</td>
<td>The outside edge / arc of the bend.</td>
</tr>
<tr>
<td>Intrados</td>
<td>The inside edge / arc of the bend.</td>
</tr>
<tr>
<td>WT</td>
<td>Wall Thickness. Specified in inches, millimeters, or wire gauge. If the wall thickness varies by 5% or more, it is advisable to specify the thickest wall dimension.</td>
</tr>
</tbody>
</table>

| CL     | Center Line of Tube. A continuous line that connect every center point of the cross section of the tube. |
| CLR    | Center Line Radius. Specified in inches, millimeters, or "D" of bend. "D" is The ratio of the centerline radius to the nominal outside diameter of the material: "D" = CLR / nominal OD. |
| Radius along the centerline of the tube to be bent is called as CLR. It is also called as Radius Of Bend. |
| Limited Bending Radius | For a given tube geometry and material properties, the minimum bend radius which can be formed successfully without any wrinkling or fracture is called the Limited Bending Radius. |
| BP     | Bending Plane. The plane that goes through the CLR and perpendicular to the direction of rotation of bend. |
| Plane Of Bend | It is the relative rotation angle between two adjacent bending planes. |
| D.O.B  | Degree of bend. Number of degrees a tube is to be bent. |
| I.S.R. | Inside radius of bend. Typically used when specifying radius to bend on rectangular and square tubes. ISR = CLR - half of O.D. |
| Wall Factor | Tube O.D. divided by wall thickness. |
| C.C.W  | Counter clock-wise or left-hand rotation of the arm swing on rotary drawtube bending machine. |
| C.W.   | Clock-wise or right-hand rotation of the arm swing on rotary drawtube bending machine. |
| C.L.H. | Centre line height. On a bending Machine, the heights from the bottom of bend die to the center of the tube. |
| D.B.B  | Distance Between Bend. It is the distance between the tangent of one bend and tangent of another bend. |
| GALLING | The transfer of one material to another caused by high pressure and friction. This occurs when stainless steel is bent with mandrel or wiper die or aluminum is bent with... |
aluminum-bronze tools. Using better lubricant, alternate material or slow arm rotation can reduce this.

| RADIAL GROWTH | After tube is bent, it will spring back and the radius of the tube will grow. It will be larger than the bend die CLR. Radial growth is greatest with CLRs greater than three times the tube OD and when bending harder material. |

V. COMPONENTS OF A ROTARY DRAW TUBE BENDING MACHINE [26, 27]

F. Bend Die

The primary tool on a rotary-draw tube-bending machine; the form against which the tube is clamped and then drawn around to produce a bend; less commonly known as the bend form or the radius die. The essential specifications of a bend die are the outside diameter and the bend radius of the tube to be bent. Fig. 8 shows a Bend Die.

G. Pressure Die

The tool that holds the tube against the bend die under pressure applied by the bending machine at the line of tangency during the rotary-draw bending process, thus creating the point of bend is called the Pressure Die. Originally the pressure die was a static die block that once crammed into position at the line of tangency did not move forward as the bend die rotated. This, of course, created considerable drag on the tube at the point of bend, which being in a plastic state, caused excessive flattening in the extrados of the bend. Fig. 9 shows the Pressure Die.

H. Wiper Die

At first blush it is a simple tool. Often the wiper is a solid block machined to fit the gap between the bend die and the back tangent of the tube to be bent. Frequently the wiper die is an uncomplicated two-piece: assembly in which the leading edge of the tool is a disposable insert. Other than size and material, there does not appear to be much to the specification of a wiper die. Fig. 10 displays the Wiper Die.

I. Clamp Die

The tool that clamps the tube with the bend die to hold it together is called the Clamp Die. The Clamp Die rotates along with the bend die and the tube. The function of the Clamp Die is to hold the tube with the Bend Die so that it rotates along with it. Clamp Die holds the front end of the tube with Bend Die.

J. Mandrel

If using a mandrel, attach the mandrel-to-mandrel rod. Advance mandrel to the correct distance past the tangent point. Make sure that the mandrel is advancing and retracting in the proper sequence. The mandrel should advance to its pre set position before the tube is inserted. After the bend is made the mandrel should retract. Further, make sure the mandrel is made from the material that is best suited for the material you are bending. For example, a aluminum-bronze mandrel should be used when bending stainless steel, titanium and other exotic materials. A hard chrome plated steel mandrel should be used when bending aluminum, steel, mild steel, and copper. Do not forget to apply lubrication. Fig. 11 shows the mandrel.
K. Ball and Ball sub-assembly

A part of the mandrel assembly, which supports the arc of the bend from flattening along the outside radius after the tubing material has passed through the point of bend. The term ball is derived from the fact that geometrically it is the center segment of a true sphere; this is why a ball is occasionally called a "sphere" or "segment". Because of its spherical geometry, as opposed to the cylindrical geometry of the mandrel nose, the ball is not very effective in setting the tubing material into a circular cross-section at the point of bend. For this reason, the mandrel should be fixture so that the entire ball, or balls, rests past the point of bend - not in the point of bend or behind it.

The flexing portion of the mandrel assembly, which consists of a series of balls linked together. This is distinguished from a mandrel sub-assembly in that a ball sub-assembly does not include a mandrel nose insert. See mandrel.

L. Barrel

That part of a link which is nested inside of the bore of a ball when assemble.

VI. GEOMETRY OF THE BENT TUBES

Geometry of tubes is the most critical parameter that affects not only the results of bends but also the selection of tooling and machinery for the process. Parameters that define the geometry of tubes include (a) tube wall thickness, \( t \), (b) outside diameter, OD, (c) centerline radius, CLR, and (d) degree of bend, DOB. Generally, the first three parameters can be converted into two dimensionless factors that are generally used in the industry to determine the feasibility of the process and the selection of the tooling. They are

- Wall factor = \( \frac{OD}{t} \) (1)
- Bend factor = \( \frac{CLR}{OD} \) (2)

In practical use, limited bending radius, LBR, is introduced to determine the bendability for a specific number of the wall factor. For different combination of tube dimensions, most of the tooling manufacturers will support tables showing the relationships between the tube geometry and the tooling selection as well as the limited bending radius.

Limited bending radius (LBR) depends on (1) the elongation property of the material and (2) the tooling design. Fig. 12 shows an example of the bendability of various aluminum materials. In the figure, it shows that the LBR decreases as the elongation capacity of the material increases when the tube OD and the wall thickness remain the same. The LBR shown in this figure were obtained from the experimental data of different aluminum pipes or tubes, which were bent on rotary draw benders with fully tooled (including mandrel and wiper die) and boosted (pressure die assist) LBR for aluminum tubes with different elongation properties can be calculated using the following empirical equations

\[
LBR = \frac{OD^2 \cdot F}{t_o} + \frac{OD \cdot \pi}{4}
\]  

And

\[
F = \pi/4S
\]  

Where, \( F = \) Slope of the lines of the LBR (see Figure 4.9)
- \( LBR = \) Limited bending radius
- \( OD = \) Tube outside diameter
- \( t_o = \) Initial tube wall thickness
- \( S = \) Minimum specified elongation as listed in the Aluminum Association Standards Data handbook

For the same tube material properties and tube dimensions, tubes can be bent into a smaller radius of bend if wiper die and ball mandrel are inserted. That is, the limited bending radius decreases when additional tooling or devices are used, such as close pitch mandrel, pressure die assist, etc.

\[
\text{Designed bending radius} > 2 \cdot OD
\]  

Fig. 12. The relationship between the wall factor and bend factor for various aluminum alloys used in draw bending. The Limited bending radius decreases as the elongation ratio of the material increases, as in [1].

For general bending design using other materials, charts that are similar to Fig. 12 may be obtained from the bending experiments (or directly from the bending tooling supplier). However, it is suggested that the radius of bend should not be less than two times of OD.

\[
\text{Designed bending radius} > 2 \cdot OD
\]  

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However, with better tooling design, correct tooling setup and proper lubrication, LBR can be reduced. A report from the automotive industry indicated that the limited bending radius could be even reduced to 0.7 of the tube outside diameter, as in [4]. However, designers should always consider the bending radius as large as possible, since the smaller the radius of bend, the higher the cost of the initial tooling and its maintenance.

Considering the tube geometry, shape of the cross-section of initial tubes is another important factor that affects the bending process. Both rectangular/square and circular cross-section tubes are widely used in the industry. Many models have been built for the analysis of the rectangular hollow tubes, as in [31]. For other special cross-section geometries, the formability of the tube can only be obtained from trial-and-error, or the finite element method/analysis (FEM/FEA).

VII. SIMULATION OF ROTARY DRAW TUBE BENDING

A finite element model of rotary draw bending is developed in ABAQUS CAE as shown in figure. The model comprises the tube; bend die, clamp die, pressure die, and wiper die.

The bend die modeled over 180 degree, thereby limiting max bend angle for the simulation tube 180 deg. The clamp die is modeled along with bend die. The bend die and clamp die, pressure die and wiper die are modeled assuming uniform clearance between the tube and the tooling. The tube was modeled as 3D deformable path of which materials are behave elastic-plastic and tools were modeled as 3D discrete rigid. The tube material is alloy steel with Young’s modulus 2.1E5 MPa. Shell elements S4R are used to model the tube geometry. S4R is a 4-node, quadrilateral stress/displacement shell element with reduced integration and a large-strain formulation. Discrete rigid elements R3D4 (a 3D rigid element with 4-node, bilinear quadrilateral) were used to mesh the geometry of tools.

Contact between various pairs of surfaces: Bend die -tube, pressure die -tube, wiper die -tube is defined using the CONTACT SURFACE TO SURFACE contact option, which allows sliding between these surfaces with coulomb friction model. This enforces a penalty function-based contact stiffness to prevent penetration of tooling by tube nodes.

Shell thickness = 1 mm
Material Density = 7360 Kg / m^3
Poisson’s Ratio = 0.30
Young's modulus = 2.1E5 N/sq. mm
Coefficient of Friction = 0.02

For different material with different ultimate tensile strength, the strain is taken correspondingly from design data book.

The tooling motion was modeled so as to be similar to the motion of bending machine. E.g., for a bend angle of 180 degrees the bend die was constrained to rotate about the bend axis with $\pi$ radians, while the pressure die was constrained to translate forward and the wiper die was constrained along all degrees of freedom, as in [23,24].

VIII. RESULTS

The simulation is obtained for 30 degrees, 45 degrees and 60 degrees for checking the Von-Mises Stress induced and then spring back is noted for various angles from 0 to 180 degrees. The various stresses induced at 180 degrees are shown in the results below.
Fig. 26. Contour of YZ-Stress (180 degree Rotation) and Spring Back

Fig. 27. Contour of ZX-Stress (180 degree Rotation) and Spring Back

Fig. 28. Contour of Effective Plastic Strain (180 degree Rotation) and Spring Back

Fig. 29. Contour of Effective Stress (180 degree Rotation) and Spring Back

Fig. 30. Contour of Min. Principle Deviatoric Stress (180 degree Rotation) and Spring Back

Fig. 31. Contour of 2\textsuperscript{nd} Principle Deviatoric Stress (180 degree Rotation) and Spring Back

Fig. 32. Contour of Max. Principle Deviatoric Stress (180 degree Rotation) and Spring Back

Fig. 33. Contour of Min. Principle Stress (180 degree Rotation) and Spring Back
IX. MATHEMATICAL CALCULATION OF SPRING BACK

M. Assumptions in Calculation

To analyze the bending process, there are some basic assumptions that are made to simplify the calculation, as in [32]. They are:

1. The initial residual stresses are neglected.
2. The tube is subjected to a pure bending moment only.
3. The bending angle is large enough so that all the characteristics will reach their constant peak values in the middle section of the bent tube. Such characteristics are normal stress, normal strain, and thickness distributions on the cross-section.
4. The plane of the cross-section remains as a flat plane after the tube is bent. That is, the normal strain distribution on the cross-section is linearly distributed with the distance from any point on the tube to the neutral plane.
5. The plastic deformation is time independent.
6. The thickness of the tube is small relative to the tube length so that the deformation due to shear stress may be neglected.
7. The cross-section remains circular as it is bent.
8. The neutral axis remains at the center of the tube, i.e., coaxial with the tube centerline.
9. No friction between the interfaces of the tube and the tooling.

N. Analytical Calculation

Consider a tube spring back process shown in Fig. 37. Assuming the tube has been bent originally from balance point D to point \( A_0 \) on circle \( O_1 \) by forming pin moving clockwise along roller. Point B was the start point of this time bending deformation. Thereafter, on the forming pin moves counterclockwise, the spring back of the tube will move from position \( A_0B \) to \( A_1B \). This process will cause a radius change, as the curve \( A_0B \) is on the circle \( O_1 \) with radius \( R_n \) and curve \( A_1B \) is on the circle \( O_2 \) with radius \( R'n \) (\( R_n < R'n \)). As the tube between point B and D has been bent before this time of bending process, the radius of curve BD will be released back to \( R_n' \) on the forming pin moving from B to D. The spring back considered coming from elastic-plastic deformation is only located along the tube between point \( A_0 \) and B. The tube between point B and D merely repeats an elastic deformation so that we will only pay attention to the tube between point \( A_0 \) and B.

Suppose the forming pin is moving cross point P on circle \( O_1 \) on spring back. The position of tube is at \( PAC \), where arc PA is located on the circle \( O_2 \) with radius \( R_n' \). When the forming pin moves from position \( P \) to B, the tube along AC will move to position \( A_1C_1 \) and get a spring back angle \( d\theta \). Meanwhile, the position of spring back tube moves to curve \( BP_1A_1C_1 \). The arc PA, because of elastic spring back, will move the circles it lies on from center \( O_1 \) to \( O_2 \). The arc BP, originally lies on circle \( O_1 \) with radius \( R_n \) and central angle \( du \), will transfer to BP1 on circle \( O_2 \) with radius \( R_n' \) and central angle \( \beta \). Thus, the point change on tube from P to \( P_1 \) will produce a tangential angle change \( d\theta' \). According to the relationship of geometry, the tube spring back angle \( d\theta \) from AC to \( A_1C_1 \) is equal to the angle variation of from \( O_1P \) to \( O_2P_1 \).
As the length of arc PB equals arc P1B, we have

\[ R_n du = R_n' \beta \]  \hspace{1cm} (6.1)

And the relationship of spring back angle \( d\theta_r \) and rolling pin moving angle \( du \) is

\[ d\theta_r = du - \beta \]  \hspace{1cm} (6.2)

Substitute (6.1) into (6.2)

\[ d\theta_r = \left( 1 - \frac{R_n}{R_n'} \right) du \]  \hspace{1cm} (6.3)

So that, the original strain before spring back is

\[ \varepsilon_o = \frac{y}{R_n} \]  \hspace{1cm} (6.5)

The final strain after spring back is

\[ \varepsilon_f = \frac{y}{R_n'} \]  \hspace{1cm} (6.6)

Where the \( R_n, R_n' \) and \( d\theta_r \) are defined in Fig. 37 and \( y \) defined in Fig. 39. The change in strain after spring back is

\[ \varepsilon_o - \varepsilon_f = y \left( \frac{1}{R_n} - \frac{1}{R_n'} \right) \]  \hspace{1cm} (6.7)

Therefore, the change in stress along tube axis direction is

\[ \Delta \sigma = E (\varepsilon_o - \varepsilon_f) = Ey \left( \frac{1}{R_n} - \frac{1}{R_n'} \right) \]  \hspace{1cm} (6.8)

According to P. Dadras (1982), the moment required to produce bending radius \( R_n \) is

\[ M = \int \sigma y dA \]  \hspace{1cm} (6.9)

The change in bending moment for complete unloading is \( \Delta M = M \). Therefore,

\[ \int_A \left[ E y \left( \frac{1}{R_n} - \frac{1}{R_n'} \right) dA \right] y = \int_A \sigma y dA \]  \hspace{1cm} (6.10)

Which reduce to

\[ \frac{1}{R_n} - \frac{1}{R_n'} = \frac{\Delta \int_A \sigma y dA}{EI} \]  \hspace{1cm} (6.11)

Where I, the area moment of inertia of the cross-section and A is the cross-section area.

In this research, the deformation of tube is a elastic-plastic bending. Along cross-section, the tube consists of two central elastic arc belts width \( C \) beside the neutral axis and two plastically zones.
remote from neutral axis, as shown in Fig. 40. According to Chakrabarty (1987), in the combined tension and torsion of a thin-walled tube, each element of the tube wall is subjected to a longitudinal stress $\sigma$ and a shear stress $\tau$. In view of von Mises criterion, the yield criterion may be expressed as

$$\sigma^2 + 3\tau^2 = S_y^2$$  \hspace{1cm} (6.12)

Where $S_y$ is the yield stress of the tube metal. On the situation of pure bending, $\tau = 0$.

$$\sigma = \frac{Er}{R_n} \sin \alpha$$  \hspace{1cm} (6.13)

Where, $R$ is the average radius of tube, $\alpha$ specify the position of the elastic/plastic boundary at any stage by angular distance from the neutral axis, shown in Fig. 40. Substitute Equ (6.13) into Equ (6.12) yield

$$\sin \alpha = \frac{R_n*S_y}{E\bar{r}}$$  \hspace{1cm} (6.14)

According to the geometry relationship in Fig. 40, if the radius of neutral axis closes to that of the centroid axis

$$\sin \alpha = \frac{C}{2r}$$  \hspace{1cm} (6.15)

Where, $\alpha$ is the average radius of tube, $\alpha$ specify the position of the elastic/plastic boundary at any stage by angular distance from the neutral axis, shown in Fig. 40. Substitute Equ (6.13) into Equ (6.12) yield

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$$\sin \alpha = \frac{C}{2r}$$  \hspace{1cm} (6.15)

Where, $\alpha$ was defined in (6.4).

As defined in Fig.4, on the situation of the tube dimension, $r_o = 0.50$ inch, $r_i = 0.4965$ inch, and radius of centroid axis $R = 3.0$ inch, the calculated radius of neutral axis $R_n = 2.990981645$ inch. It is reasonable to consider the neutral axis and centroid axis are coinciding on each other.

Where, $A_e$ and $A_p$ are the elastic and plastic areas. $\varepsilon$ was defined in (6.4).

Consider this spring back is coming from the plastic deformation first. On getting $R_n'$ from (6.19) and substituting $R_n'$ into (6.3) to find $\theta$, the bending angle $\alpha$ need to be integrated within this times angle increment but all bending angle.

**O. Derivation for Calculation of Springback**

As shown in above Fig. 41, when the external machine load is release, elastic spring back will decrease the total bending angle and increase the bending radius simultaneously. In analytical calculations, it is assumed that the total bending length in the deformation zone remains the same at all time, as in [31]. That is
So

\[ \Delta \theta = \theta_L - \theta_U \]  

And the curvature changes due to the radial growth is defined as

\[ \Delta K = K_L - K_U \]  

Where,

\[ \theta_L = \text{Total bending angle in loading condition} \]
\[ \theta_U = \text{Total bending angle in unloading condition} \]
\[ K_L = \text{Curvature of bend in loading condition} \]
\[ K_U = \text{Curvature of bend in unloading condition} \]

So,

\[ K_U = K_L - \Delta K \]  

From the moment curvature Diagram the curvature changes due to the spring back, that can be calculated as follows,

\[ \Delta K = \frac{M_L}{(dM/dK)} \]  

Where dM/dK is the slope of the M K relationship in the elastic region that is given by the following equation,

\[ \frac{dM}{dK} = E \cdot I \]  

I = the Moment of Inertia of the tube (\( I = \pi r^4 t_0 \))

Since \( K_U = 1 / RU \) by substituting (6.24) into (6.21)

\[ \theta_U = \theta_L R_L (K_L - \Delta K) \]  

By substituting (6.25) and (6.26) into (6.27) the final bending angle \( \theta_U \) after spring-back is

\[ \theta_U = \theta_L (1 - \frac{R_L M_L}{EI}) \]  

Where,

\[ \theta_L = \text{Total bending angle in loading condition} \]
\[ \theta_U = \text{Total bending angle in unloading condition} \]
\[ R_L = \text{Bending radius in loading condition} \]
\[ M_L = \text{Bending Moment applied in loading condition by Machine} \]
\[ E = \text{Modulus of Elasticity} \]
\[ I = \text{Moment of inertia of the tube section (} I = \pi r^4 t_0 \) \]

X. CALCULATION OF SPRING BACK

P. CASE 1

Given

Tube dimension: OD 50 mm X Thickness 06 mm
Material: ST52 NPC GRADE 6

Material Properties

Modulus of elasticity = 2.1x10^5 N/mm^2
Poisson's ratio = 0.3
Density = 9.8 g/cm^3
Yield stress = 375.94 N/mm^2
Strength coefficient = 1.44 x 10^-3
Strength strain coefficient = 0.51
U.T.S. = 524.57 N/mm^2
% Elongation = 27.1 %
Modulus of rigidity = 0.790 x 10^5 N/mm^2

Calculation Of Bend Radius:

\% Elongation= \( \frac{[(R+2r)/(R+r)]-1 \times 100}{27.1 = \frac{[(R+25)/(R+50)]-1 \times 100}{R= 68 \text{ mm}} \)

Calculation Of Moment of Inertia:

\[ I = \pi \cdot 0.25 \cdot (0.5^4 - 0.5^4) = 204.442 \times 10^3 \text{ mm}^4 \]

By using formula (3)

Spring Back for Different Bending Angles:
TABLE II. EXPERIMENTAL AND CALCULATED SPRING BACK ANGLES FOR CASE 1

<table>
<thead>
<tr>
<th>Bending Angles (Degrees)</th>
<th>Spring Back (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
</tr>
<tr>
<td>05</td>
<td>0.2321</td>
</tr>
<tr>
<td>10</td>
<td>0.2982</td>
</tr>
<tr>
<td>15</td>
<td>0.3826</td>
</tr>
<tr>
<td>30</td>
<td>0.7653</td>
</tr>
<tr>
<td>45</td>
<td>1.1479</td>
</tr>
<tr>
<td>60</td>
<td>1.5241</td>
</tr>
<tr>
<td>90</td>
<td>2.2959</td>
</tr>
<tr>
<td>120</td>
<td>2.5214</td>
</tr>
<tr>
<td>150</td>
<td>2.8719</td>
</tr>
<tr>
<td>180</td>
<td>3.1415</td>
</tr>
</tbody>
</table>

Fig. 41. Graph of Bend Angle v/s Spring Back Angle for CASE 1

Q. CASE 2

Given:
Tube dimension: OD 30mm X Thickness 03mm
Material: ST52 NPC GRADE 6
Material Properties:
- Modulus of elasticity = 2.1 x 10^5 N/mm²
- Poisson's ratio = 0.3
- Density = 9.8 g/cm³
- Yield stress = 375.94 N/mm²
- Strength coefficient = 1.44 x 10⁻⁴
- Strength strain coefficient = 0.51
- U.T.S. = 524.57 N/mm²
- % Elongation = 27.1 %
- Modulus of rigidity = 0.790 x 10⁵

Calculation Of Bend Radius:
\[ \% \text{Elongation} = \frac{(R+2r)/(R+r)-1}{R+15} \times 100 \]
\[ 27.1 = \frac{(R+30)/(R+15)-1}{R+15} \times 100 \]
\[ R = 15 \text{mm} \]

Calculation of Moment of Inertia:
\[ I = \pi \times 0.25 \times (r_o^4 - r_i^4) = 23.474 \times 10^3 \text{ mm}^4 \]
By using formula (3)

Spring Back for Different Bending Angles:

TABLE III. EXPERIMENTAL AND CALCULATED SPRING BACK ANGLES FOR CASE 2

<table>
<thead>
<tr>
<th>Bending angles (Degrees)</th>
<th>Spring back (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
</tr>
<tr>
<td>05</td>
<td>0.1301</td>
</tr>
<tr>
<td>10</td>
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<td>30</td>
<td>0.2315</td>
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<td>45</td>
<td>0.3459</td>
</tr>
<tr>
<td>60</td>
<td>0.5781</td>
</tr>
<tr>
<td>90</td>
<td>0.7241</td>
</tr>
<tr>
<td>120</td>
<td>0.8125</td>
</tr>
<tr>
<td>150</td>
<td>1.1459</td>
</tr>
<tr>
<td>180</td>
<td>1.2122</td>
</tr>
</tbody>
</table>

Fig. 42. Graph of Bend Angle v/s Spring Back Angle for CASE 2
XI. RESULTS

Fig. 19. Fig. 20. Fig. 21. Shows the variation of Von-Mises stress for a tube bent at 30 degree, 45 degree and 60 degree

The maximum stress observed for

1. 30 degree bend angle is $3.112 \times 10^4$ N/mm$^2$
2. 45 degree bend angle is $4.447 \times 10^4$ N/mm$^2$
3. 60 degree bend angle is $4.755 \times 10^4$ N/mm$^2$

Fig. 22 - Fig. 36. Shows the different stresses and strains such as stress along X, Y & Z-axis, plane stress i.e. $\sigma_{xy}$, $\sigma_{yz}$, $\sigma_{zx}$, plastic strain, effective stress, Principal stresses & Principal Deviatory stresses for the tube bend at 180 degree.

The maximum stress observed for 180-degree bend angle is as follows

1. $\sigma_x = 0.37248$ KN/mm$^2$, $\sigma_y = 0.49703$ N/mm$^2$, $\sigma_z = 0.39056$ N/mm$^2$,
2. $\sigma_{xy} = 0.175581$ KN/mm$^2$, $\sigma_{yz} = 0.178576$ KN/mm$^2$, $\sigma_{zx} = 0.20722$ KN/mm$^2$.
3. Effective Plastic Strain = 0.18099, Effective Stress = 0.474456 KN/mm$^2$, Min.
4. Principal Deviatory Stress = $1.2236 \times 10^{-6}$ KN/mm$^2$, 2nd Principal Deviatory Stress = 0.123631 KN/mm$^2$, Max Principal deviatory Stress = 0.31241 KN/mm$^2$,
5. Min. Principal Stress = $4.0603 \times 10^{-7}$ KN/mm$^2$, Max. Principal Stress = 0.510398 KN/mm$^2$.

XII. CONCLUSION

After all work of literature review, industry survey analytical equation derivation FE simulations and experimental data my thesis has achieved the following conclusion,

1. During the simulation it was observed that at 30 degrees bend angle the stress reached the yield stress in the fiber from outside and inside of curvature. Between these zones there is middle zone in which the strains are still in elastic field.
2. Penetration of bend die is not observed inside the tube for 30 degree and 45 degree rotation of the die. But for 60 degrees rotation of the die penetration of the bend die occurs.
3. The thesis gives the technology in tube bending process.
4. Analytical method / Equations in calculating Spring Back are developed.
5. Techniques to run simulation for rotary draw bending process by using ABAQUS are developed.
6. Simulation models with tube dimension are proved to have good agreement with the analytical methods
7. As angle of rotation goes on increasing, ALLAE also goes on increasing from 400 to 1000 to 1500 for the same time of simulation.
8. Folding of tube at certain areas is observed due to absence of mandrel. This can be eliminated by using
   - Fixed mandrel or
   - Flexible mandrel or
   - By using internal pressure (pressurized fluid in the tube)

XIII. REFERENCES


